we desire at all proper times to hold up to the censure and watchfulness of the people of the United States, — has in later times been shamed into occasional acts of exploration along the Arctic Sea. It professes to have finished that, which Parry, Ross, and Franklin had all but finished. Messrs. Dease and Simpson, of the Hudson's Bay Company, have recently explored the little there was left of unknown betwixt the mouth of Mackenzie's River and Behring's Strait. And we may now aver, — *There is a Strait of Anian.* That is to say, there is a water communication (though more or less obstructed by ice) from the Atlantic to the Pacific, along the arctic side of North America. That being the fact, it might be well, as a matter of historical curiosity, to reconsider the stories of Maldonado, De Fonte, Urdaneta, and Ladrillero, and to compare them with modern observation, so as to judge how far they may thus appear, any of them, to have been founded on actual discovery and knowledge, or to be pure fable. The result of this might be to restore merited honor to another Juan de Fuca.

ART. IV. — 1. *Traité de Mécanique Céleste.* Par P. S. LAPLACE, Membre de l'Institut National de France et du Bureau des Longitudes. Tome Premier, pp. 368, et Tome Second, pp. 382. An VII. 4to.

2. *Traité de Mécanique Céleste.* Par P. S. LAPLACE, Membre du Sénat Conservateur, de l'Institut National et du Bureau des Longitudes de France ; des Sociétés Royales de Londres et de Gottingue, des Académies des Sciences de Russie, de Danemark, d'Italie, etc. Tome Troisième. Paris, An XI. 1802. pp. 303.

3. *Traité de Mécanique Céleste.* Par M. LAPLACE, Chancelier du Sénat Conservateur, Grand-Officer de la Légion d'Honneur, Membre de l'Institut et du Bureau des Longitudes de France ; des Sociétés Royales de Londres et de Gottingue ; des Académies des Sciences de Russie, de Danemark, d'Italie, etc. Tome Quatrième. An XIII. = 1805. pp. 347.

4. *Traité de Mécanique Céleste.* Par M. LE MARQUIS DE LAPLACE, Pair de France ; Grand Croix de la Légion d'Honneur ; l'un des Quarante de l'Académie Fran-

çaise; Membre du Bureau des Longitudes de France; des Sociétés Royales de Londres et de Gottingue; des Académies des Sciences de Russie, de Danemark, de Suède, de Prusse, des Pays-Bas, d'Italie, de Boston, etc. Tome Cinquième. 1825. pp. 419.

5. *Mécanique Céleste.* By the MARQUIS DE LA PLACE, Peer of France; Grand Cross of the Legion of Honor; Member of the French Academy; of the Academy of Sciences at Paris; of the Board of Longitude of France; of the Royal Societies of London and Göttingen; of the Academies of Sciences of Russia, Denmark, Sweden, Prussia, Holland, and Italy; Member of the American Academy of Arts and Sciences, &c. Translated, with a Commentary, by NATHANIEL BOWDITCH, LL. D., Fellow of the Royal Societies of London, Edinburgh, and Dublin; of the Astronomical Society of London; of the Philosophical Society, held at Philadelphia; of the American Academy of Arts and Sciences, etc. Volume I. 1829. pp. 746. Volume II. 1832. pp. 990. Volume III. 1834. pp. 1017. Volume IV. pp. 1018.
6. *A Discourse on the Life and Character of the Hon. Nathaniel Bowditch, LL. D., F. R. S., delivered in the Church on Church Green, March* 25, 1838. By ALEXANDER YOUNG. Boston: Little & Brown. 8vo. pp. 120.
7. *An Eulogy on the Life and Character of Nathaniel Bowditch, LL. D., F. R. S., delivered at the Request of the Corporation of the City of Salem, May* 24, 1838. By DANIEL APPLETON WHITE. Salem: 8vo. pp. 72.
8. *Eulogy on Nathaniel Bowditch, LL. D., President of the American Academy of Arts and Sciences; including an Analysis of his Scientific Publications. Delivered before the Academy, May* 29, 1838. By JOHN PICKERING, *Corresponding Secretary of the Academy.* Boston: Little & Brown. 8vo. pp. 101.

COMPARED with the infinite variety and extent of physical phenomena, the domains of the science of quantity would, at first sight, seem confined to very narrow limits. But when we consider, that mathematics treat of all forms and motions; when we find the sweetest tones and the brightest colors, the lightning and the rainbow, heat and cold, and the very winds

and waves subject to the strictest laws of motion ; when, in short, we learn, that the whole world is bound together upon mechanical principles ; we must concede that the ocean, upon which the mathematician has launched his ship, is as unbounded as the material universe. But, of all sciences, the one the best fitted for the application and advancement of geometry is Astronomy ; and the phenomena of the firmament, as developed by the geometer, are so glorious and so sublime, as to realize the fabled music of the spheres. Here, however, we must stop, and not trench upon the empire of fancy. It is for the man of science to calculate the harmony of the heavens with his slate and pencil, and he may not presume to sing it with the harp of the bard ; it is not his to gaze, to admire and wonder, but to observe with care, to know and comprehend. As the sailor is not the poet of the sea, neither is the mathematician the poet of the sky ; but, like a rapid and impetuous torrent, the depth of his channel deprives him of the view of the fertile fields and splendid scenery, through which he is hurrying to his sole object, the sea of truth.

Mournful is it to think, that national prejudices could ever intrude themselves into the council-chamber of philosophy. But so it is. Though France has, for many years, been the land of mathematicians, her eminence has not taught her magnanimity. Her neighbours, England, Germany, and Italy, have complained of her eagerness to appropriate their labors, and of her reluctance to acknowledge their merits ; and more than once has she cast shafts of envy even at the immortal legislator of the physical universe. The historian of astronomy, Delambre, of whom she is rightly proud, did indeed allow, that Newton stood alone, above all other astronomers, and called him "cet homme unique"; but, nevertheless, he dwelt much upon the exaggerated homage the English paid him, upon the steps which previous mathematicians had taken in anticipation of his discoveries, and upon the imperfections of the "Principia." He says of the lunar theory, that he had been sometimes tempted to suspect, that some of the results were not derived, as they professed to be, from theory, but from the examination of observations ; confessing, that he would not have ventured upon such an insinuation, if it had not been before made by Clairaut. He is for giving Bouilland the honor of discovering the law of gravitation, and Newton that

of demonstrating it ; and even this share of the glory Newton must divide with his good fortune. Because he had received from Kepler the laws of the planetary motions, and from Huygens those of motion in general ; because an accurate measure of the earth had been recently made ; and, more than all, because he had the analysis which he himself created ; Delambre would conclude, with Lagrange, that, even if Newton were the greatest of mathematicians and natural philosophers, he was also the most fortunate. "For," he says, "a world to be explained, and materials ready for its explanation, are to be met with but once ; to meet with them was Newton's good fortune ; he knew how to profit by it ; there lies his glory."

These conclusions, not unsupported by facts, but characterized by a tone of jealousy unworthy of the amiable and illustrious Lagrange, were repeated in the "*Système du Monde*" ; as if such good fortune were not always the lot of genius. Galileo found it in the almost accidental invention of the telescope ; Kepler, in the friendship of Tycho Brahe, without whose observations he might never have left a wilderness of fanciful speculations, in which the present fashionable philosophy would have rioted, to reap his glorious harvest from the fruitful fields of inductive science ; and Laplace himself found it in the ready assistance and coöperation of the first observers and calculators of Europe. Nature, indeed, seems, when casting the lot of her great men, to load her dice with peculiar care ; and her favorite sons are ever born at the most fortunate epochs. But the author of the "Optics," the "Fluxions," and the "Principia," was, in no respect, the genius of accident ; his thrice-won immortality towered far above his good fortune ; and the striking fact must not be forgotten, that precisely the same good fortune was that of his noble rival, Leibnitz, who had likewise created the very same analysis. But, far from knowing how to profit by it, as Newton had done, he even opposed the "Principia" and the law of gravitation with all his might, was joined in his opposition by the first geometers of the age, and, in the words of Biot, "it was half a century before the great truth, contained and demonstrated in the 'Principia,' was, I will not say, followed and developed, but before it was even comprehended by the majority of the learned."

The recent discovery of some old papers has led to singu-

lar discussions regarding the reputation of Newton ; and some have pretended, that, because the observations were essential to the calculations of the lunar theory, the observer Flamsteed is entitled to a large share of Newton's glory. With equal justice the organ-blower might lay claim to the merit of the music of Mozart. The mere observer is only a higher order of mechanic ; his "Historia Cœlestis" displays only his untiring industry, and the acuteness of his senses ; he is but the hand of astronomy, and may not wear the crown of glory fitted to the head, which combines his observations into an harmonious theory.

But the private character of Newton has suffered more severely from these developements ; and the snails of literature have been most industrious in defacing the reputation once so bright and unsullied. However much their writings may please a world, but too eager to prune all men down to the same level, the generous mind will turn from them with disgust to Laplace's last labor, which, purporting to be a history of mathematical astronomy, is, in reality, the noblest and truest eulogy upon him who laid its foundations.

The first division of this volume, treating of the Mathematical Theory of the Earth's Figure, begins with the fact, that Newton founded this theory ; and concludes an accurate account of Newton's labors upon it with the remark, that, notwithstanding several hypotheses, one of which is contrary to later observations, this step must be regarded as a prodigious one, considering the importance and novelty of the propositions established by its author, and the extreme difficulty of the subject. We must here give another remark of Laplace, made upon the lunar theory, — that these hypotheses may be allowed to inventors in such profound researches ; and we may add, that Newton's instinct in such hypotheses was almost unerring, was altogether unrivalled, and seemed to border upon the divine. In the book upon the Attraction of Spheres, and the Motions of Elastic Fluids, Newton is said to have been the first to consider the attraction of spherical bodies, and his theory of sound is pronounced to be a monument of his genius. This theory had been objected to by Lagrange, in the "Turin Miscellany," on account of the alleged unsoundness and paradoxical nature of its reasoning ; and his objections have been quoted and repeated by later mathematicians, among whom we blush to write the

name of an Englishman, and so eminent a philosopher as the younger Herschel. These geometers seem never to have examined either the original theory, or the attack upon it; and must have been ignorant of the fact, that, with a magnanimity of which few are capable, Lagrange had, in the "Memoirs of the Berlin Academy," retracted his objections, and admitted the principles of Newton's theory to be indisputably correct. The arguments against the theory, far from showing its inaccuracy, were, indeed, the most conclusive demonstration of its generality, and only proved, that Newton needed not to limit it to a particular case, and should not have inferred this case to be the one of Nature, because it satisfied his theory.

In his next book, upon the Oscillations of the Fluids which cover the Planets, La Place says, that Newton first gave the true theory of the ebb and flow of the sea, attaching it to his great principle of universal gravitation. Euler, the father of modern analysis, concluded the method of Newton to be entirely erroneous, and not even to have approached the truth; but Laplace points out the cause of the difference between the results of these two profound geometers, and shows that Newton's theory, instead of deserving Euler's reproaches, was most admirable for its ingenuity.

In the chapter upon the Precession of the Equinoxes, the Newtonian solution of this most intricate of problems is carefully analyzed, and proved to have but one defect, which, though "a radical one, is quite excusable in a first inventor, and would probably have been corrected, if Newton's other occupations had allowed him to pay a closer attention to the discoveries of the continental mathematicians."

The honor of discovering the law of gravitation, and thus laying the corner-stone of the celestial mechanics, is, in this volume, given most unreservedly to the author of the "Principia," and without any of those qualifying remarks upon his good fortune, which had intruded themselves into the "*Système du Monde.*"

Newton's researches into the theory of the moon, which had met with so sad a reception from Clairaut and Delambre, are unhesitatingly pronounced, by Laplace, to be one of the most profound parts of his admirable work; and a portion of them appeared to him so remarkable for ingenuity, that he devoted a chapter to the task of translating this portion into the language of modern analysis.

Such is a brief review of Laplace's notices of Newton's labors; and they show, that, in all the profound discussions of the "*Mécanique Céleste*," the first steps had been taken in the "Principia." But La Place is not the only French philosopher who has risen above national prejudices, and cheerfully acknowledged all the greatness of Newton. Biot, also, calling the author of the "Principia" "the creator of natural philosophy," regarded him, moreover, as having founded the principles of the Mechanics of Chemistry, by referring its combinations to atomic attractions and repulsions, and by taking the boldest, happiest, and most original views of the composition of bodies. The evil genius even of Newton gained, however, one victory over his good fortune, and deprived the world for ever of his more sublime speculations upon this vast subject. By an unlucky accident, the labor of years was burned to cinders, the mighty soul of Newton was agitated with an emotion, which his physical powers could not resist, and from which, it is doubtful if they ever recovered, and his dog Diamond achieved for himself a share in his master's immortality. Were it not for this accident, we cannot conjecture how high the monument of his genius would have aspired, and how many later discoveries he would have anticipated; but enough remains to justify and command the homage of every man of science, whether English or French, and to vindicate his epitaph, "Congratulentur sibi mortales tale tantumque exstitisse humani generis decus."

Although the fifth volume of the "*Mécanique Céleste*" was published more than twenty years after the preceding ones, it is, from its historical character, the real preface to the whole work. It will be adopted as the text for the present article, and its division into subjects will be closely followed. Returning, then, to its first chapter upon "The Figure and Rotation of the Earth," we will notice more particularly, the hypotheses assumed by Newton, without demonstration, in order to evade the difficulties of the calculus. He supposed the earth to be an ellipsoid of revolution, and to be homogeneous; and gravity to increase from the equator to the poles, in proportion to the square of the sine of the latitude. The first of these hypotheses is quite a natural one, since the ellipse is the most simple of ovals; but the last, though neat in its form, could, by no means, have been obvious to the

original inquirer. Both of these have, however, been confirmed by the successive demonstrations of Clairaut, Maclaurin, D'Alembert, Legendre, and La Place; but La Place has proved, that the second hypothesis is inconsistent with known phenomena, and that the earth is not homogeneous. The researches upon this subject assume the earth to have attained its present form while in a fluid state, or at least while covered with a fluid, and are, therefore, based upon the principles of hydrostatics.

The general principle of the equilibrium of fluids was unknown to Newton, and he adopted an imperfect one, which was, however, when combined with his hypotheses, sufficient for his purpose. He supposed a canal, consisting of two branches, to be drawn from the centre of the earth, the one branch to the pole and the other to the equator; and inferred, that the pressures upon the bases of the branches must be equal, because the fluid contained in the canal must be at rest of itself, independently of the surrounding fluid. Huygens, about the same time, introduced the condition, that the surface of the fluid must be upon a level, that is, must be perpendicular to the direction of a falling body; which was united by Bouguer with that of Newton. But the combination was not sufficient for all cases; and Maclaurin, generalizing the idea of Newton, was led to the great principle, that the pressures upon the bases of all the straight canals drawn from *any* point of the fluid to its surface must be equal. Clairaut's more famous and more recent principle, which has been adopted by almost all mathematicians, that, if an oval canal be drawn, of any curve whatever, the fluid contained within it must be at rest, independently of the other parts of the fluid, may easily be shown to be but a slight and unimportant generalization of that of Maclaurin; but it deserves its reputation, since its author, by translating it into algebraical language, deduced from it the equations of the equilibrium of fluids; "a discovery which," says Lagrange, "has changed the face of hydrostatics, and made it a new science." These equations can, however, be deduced from the more fundamental principle, which is almost identical with Maclaurin's, that a fluid presses equally in every direction; but this principle itself has been traced by Lagrange back to its source, and it flows directly and necessarily from the definition of a fluid, that its particles move by each other with perfect facility. That a fluid will be at rest when each particle is pressed

equally in every direction is, however, too obvious a truth to have escaped the acuteness of antiquity. We find it involved in the proposition of Archimedes, that the less pressed particles will force away those which are more so; and when he adds, that the pressure upon a particle is equal to the weight of all the column above it, we feel, that he only needed the language of modern analysis to have perfected his theory of hydrostatics.

Clairaut's condition is so intimately united with this fundamental principle, that its completeness would seem no more subject to doubt than its evident necessity; it has, however, been doubted by that most accomplished geometer, Ivory, who has proposed another condition, which he thought must be added to it. The propriety of this addition has been much discussed by mathematicians, and successfully resisted by Poisson, who has pointed out several cases in which it would lead to erroneous results. Bowditch has, also, tested it, in a single instance, and, by a neat diagram, exposed its fallacy to a mere glance of the eye. Since the publication of Bowditch's volume, and without any allusion to it, Ivory has, however, somewhat changed his ground, and struck out that feature of his principle which was the peculiar object of attack. The paper, in which Ivory distinctly confessed the exceptionable point of his previous statement, is published in the "London Philosophical Transactions" for 1834, and stands altogether alone as a specimen of geometrical sophistry. Never did a demagogue exert himself more to break up the foundations of society, and reduce it to a chaos in which the lowest might hope to rise, than Ivory has done, by a most artful inaccuracy of expression and looseness of reasoning, to confuse our ideas upon the principles of hydrostatics, and thus save his crude speculations from a merited censure, if not obtain for them a portion of respect and admiration. The contempt which he here expresses for the labors of other geometers is in curious contrast with the praises bestowed in an earlier paper. To grasp and expose the errors of his obscure argument were no easy task. Near the end of his paper, he remarks, that, when a homogeneous fluid spheroid, rotating upon an axis, is in equilibrium, any similar spheroid, taken within it, and having the same axis, must also be in equilibrium, independently of the surrounding stratum. "It would," as he says, "be superfluous to repeat the de-

monstration of this proposition, as it is attended with no difficulty, and has not been contested. And because the fluid of the inner spheroid is separately in equilibrium, with respect to the centrifugal force of its particles, and the attraction of its mass, it must likewise be in equilibrium with respect to the other forces that act upon it ; for, were it not so, the whole body of the fluid would not be in equilibrium." To this point we agree with him, but we cannot admit, that "the other forces" are only the attraction of the exterior stratum, and thus leave out of view the pressure of this stratum, which seems to us to vary upon the different points of the surface of the internal spheroid, and not to be everywhere the same, as is shown by Ivory to result from his principle. The error of this result is, indeed, no less evident from Bowditch's diagram, than the fallacy which it was intended to expose. It would follow from it, that, if the earth were a perfectly stationary, homogeneous fluid, and if a solid sphere of the same density, and of a mile in diameter, were to be let down into it, until the upper surface just touched that of the earth, the pressure upon the sphere would be everywhere the same, and the point a mile below the surface of the earth, and which bears the weight of a column of fluid a mile high, would be no more pressed than the upper point, which is not pressed at all.

Clairaut applied his calculations to the case of a solid elliptical nucleus, covered entirely with a fluid, which Laplace afterward generalizing, extended his researches to a nucleus of any nearly spherical form ; and in his last volume he has gone still further, and, surmounting every obstacle, has shown that all the irregularities of continents and seas may be included in the analysis. "In thus," he says, "approaching Nature, the causes of several important phenomena, presented by Natural History and Geology, may be discovered, and great light be shed over these two sciences."

In order to verify and complete these profound investigations, it is important to compare the oblateness of the earth deduced from them, with that indicated by observations. The most obvious method of determining the figure of the earth from observation, is, to measure its curvature in different places, that is, the lengths of its degrees. From a comparison of such measurements, La Place concluded, that the hypothesis of an elliptical meridian was but an imperfect ap-

proximation to the truth. This method is, however, less to be relied upon than any other which has been used, because slight deviations from the regular figure are much magnified by it, and the measures are liable to very great inaccuracies. Of the seven measurements, indeed, collected by La Place, and which would seem to have been made most carefully and at great expense, four have been rejected, upon further examination, as unworthy of credit, another has been corrected, and only two have since been added to supply their place. Nor is this defect of observation so very wonderful, when we reflect, that, for man to measure the earth is scarcely less disproportioned to his physical dimensions, than for the ant to measure the mountain upon which he has raised his little hill.

Retaining, then, only these five of the most extensive and accurate observations, Airy has computed the earth's form, supposing it not to be precisely elliptical ; but, from the same observations, Bowditch has, by an improved method of his own, obtained a far more satisfactory result. Bowditch has also applied his method to these observations, supposing the earth to be elliptical, and has still obtained a figure "more conformable to them than the irregular figure resulting from Airy's calculation," and sufficiently agreeing with them to prove, that the elliptical hypothesis should by no means be neglected. The oblateness, thus obtained, is about one three-hundredth, which is that generally adopted by astronomers ; and Bowditch's results are fully sustained by the calculations of Ivory.

Another method of measuring the actual oblateness is taken from the observations of the seconds' pendulum. The value, obtained from them by La Place, was much too small, but has been considerably augmented by Bowditch's corrections of some errors in his calculations ; and from later and better observations, Bowditch and Ivory have both deduced a value, which does not differ much from one three-hundredth. Even these observations of the pendulum are not, however, to be relied upon with any great degree of confidence ; but, as Bowditch justly remarks, "instead of being dissatisfied with this result, we ought to feel some degree of surprise, that the difference between the polar and the equatorial radius of the earth can be determined within a fraction of a mile, by means of the very small excess of the polar, over the equatorial pendulum, which may be considered as a

base line of less than a quarter of an inch in length," which is only about one three-millionth part of the length to be ascertained, or one thousand-millionth part of the radius of our globe. How surprising is it, then, that La Place has been able to deduce not only the form of the earth, but even, in a degree, its internal structure, from the observed variations in the length of the pendulum; and to prove, most conclusively, that the earth is not homogeneous, that its density continually increases from its surface to its centre, and that its layers of different densities are disposed regularly about its centre, in a form almost elliptical!

In determining the earth's figure, either from the length of the degree or that of the pendulum, different measurements are not found to lead to the same result; and it is, therefore, necessary to adopt that, which most nearly satisfies the observations. Various methods have been proposed for selecting the preferable ellipse, founded upon different ideas of probability. The most natural one is, perhaps, to seek that ellipse, in which the greatest error is as small as possible; but a single instance will exhibit its unsatisfactoriness. If, of one hundred observations, ninety-nine coincide in the same result, we should find, that the minimum of the greatest error corresponded to an oblateness which was half way between the result of the single observation and that of all the others; and one observation would, in this method, be worth full as much as the remaining ninety-nine. A more philosophical method is that of the least squares; and a still better one is that proposed by Boscovich, and adopted by La Place; it requires the sum of all the errors to be a minimum, and also the sum of the positive errors to equal that of the negative errors. Bowditch has, however, given a correction of the method of the least squares, which seems to raise it even above that of Boscovich; and we have already alluded to the entire success which he met with in its applications. He observed, that, in the common method of least squares, an arc of ten degrees has one hundred times the influence of an arc of one degree. He then continues;

"This is unreasonable; for an arc of ten degrees, measured by one person, with the same instruments, and by the same method, is liable to the imperfection of the peculiar manner of observation of that single observer, and to the errors of *one* set of instruments; and it cannot be doubted, that *ten* consecutive

degrees, measured by *ten* different persons, of equal skill and carefulness in observing, each being furnished with instruments of the same completeness and accuracy, would be at least as satisfactory as ten consecutive degrees, measured by only one of these observers, with the same single set of instruments, notwithstanding the advantage, in this last measure, of requiring only two observations of the latitude throughout the whole arc. We shall therefore assume as a principle, that, *in the application of the method of the least squares to geodetical measures, we must suppose any arc of the length of* i *degrees to have the same weight as* i *single degrees, measured separately.*" — Vol. II. p. 435.

A third method of obtaining the oblateness, which is considered by La Place superior to the others, is derived from observed irregularities in the motion of the moon, so that the astronomer, without leaving his observatory, is able to determine the earth's figure with unrivalled accuracy.

The form of the earth, thus deduced from observation, hardly differs from that which it would have, if the ocean were taken from it, and its surface reduced to a fluid state. Hence, and from the small density of the sea compared with the earth, La Place inferred, that the average depth of the ocean must be small; and his inference is confirmed by the small height of the tides. The regularity with which the different strata are disposed about its centre, has also led him to the conclusion, generally maintained by geologists, that the whole earth was once fluid. He has proved, that it is not necessary to suppose the interior shells to be chemically different in their component parts, for that all known phenomena would be equally well satisfied, if they were of the same compressible substance.

In all astronomical observations, the length of the day and the latitude of the place are assumed to be invariable; so that it is of the highest practical importance to prove, that the earth is for ever rotating upon the same axis with perfect uniformity. Now geometers have long known, that every solid body has three principal axes of rotation, about any one of which, if it once turns, it can never cease to turn with unchanging velocity, unless from the powerful influence of some foreign force. One of the earth's principal axes would be its present axis, if it were entirely solid; but it would seem as if the sea, with its perpetual oscillations, must affect the regularity of the rotation. A profound analysis, however, led

La Place to the directly opposite conclusion, that, if the earth's rotation were slightly disturbed, and the position of its axis a little changed, the mobility of the ocean would soon bring it back to a permanent state of rotation about a fixed axis. He has also proved, that the attractions of the sun, moon, and planets, that volcanoes and earthquakes, that winds, rivers, and currents of the sea, can never have any sensible effect upon the place of the poles, or the duration of the day; and his conclusions are confirmed by records of ancient eclipses, from which it appears, that the length of the day cannot have changed, since the time of Hipparchus, by the hundredth part of a second.

Such an unchanged rate of rotation seems almost irreconcilable with the hypothesis, that the earth's primitive fluidity arose from its melting heat, that it is now constantly cooling, and therefore lessening its diameter, and increasing the velocity with which it turns upon its axis. But this hypothesis is powerfully sustained by thermometrical observations made in mines, which indicate a rapid increase of heat with that of depth, and demonstrate, independent of any hypothesis, that the earth is moving in a space much below its own temperature, so that it must be growing colder. Having, then, expressed the continual loss of heat of a cooling globe by a series of terms, which are of such a form as to disappear, one after another, in the course of time, La Place supposes our planet to have arrived at that state in which only one of these terms remains, of any sensible magnitude. This supposition may, as he says, be far from the truth, but it answers well enough to exhibit the slow rate at which the earth's temperature is diminishing, and the slight effect thus produced upon the length of the day, amounting to less than one three-hundredth of a second since the time of Hipparchus, and consequently, not to be detected in such observations as have been handed down to us. Another most remarkable result of La Place's calculation is, that the internal heat of the earth does not affect the temperature of its surface by more than one-fifth of a degree; and, therefore, when it is entirely cooled, its climates will not be the less adapted to the support of all the organized beings at present upon it.

Intimately connected with its figure is the law of the earth's attraction upon a point, either of its surface, or at a distance from it; and a concise history of the labors of geometers upon this subject is thus given by Bowditch.

"The computation of the attraction of an ellipsoid, partially treated of by Newton, in his 'Principia,' was extended by Mc Laurin, in a geometrical solution remarkable for its elegance and simplicity, to the determination of the whole attraction upon any point, within or upon the surface of the ellipsoid of revolution. The same result was afterwards obtained by Lagrange, in an analytical form, by a change of coördinates. Legendre extended the investigation so as to embrace all points, whether within or without the surface of the ellipsoid. Finally La Place obtained the general attraction of any ellipsoid, in all cases, whether the principal axes were equal or unequal, and upon any point without or within the surface. Since the publication of his method, a great improvement has been made by Ivory, in which the computation of the attraction of any ellipsoid upon an external point is reduced, by his analysis, to the much more simple case, of finding the attraction of another ellipsoid, upon an internal point, or upon a point situated in the surface of the ellipsoid. Ivory has also treated the subject in a very elegant geometrical manner, in the *Encyclopædia Britannica*, under the article *Attraction*." — Vol. II. p. 11.

Most of the high questions of astronomy have stood far above the powers of Geometry; but this problem of calculating the attractions of an ellipsoid is peculiarly adapted to synthetical methods of research, and Geometry has for once, under the colors of England, striven with success against the mightier genius of analysis. The results of Maclaurin's geometrical solution were not reached by Algebra till the succeeding generation; and, notwithstanding the further advances of later mathematicians, the two most ingenious solutions which have yet appeared, offspring of the same mind, are of equal beauty; and, though one is geometrical, the other algebraical, neither can claim a decided superiority.

The method of calculation which La Place has used throughout his analysis of the figures and attractions of the planets, exhibit most strikingly his mighty power in wielding the weapons of Algebra, and forcing his way directly to his point with irresistible energy. They occupy the third book of the "*Mécanique Céleste*," which is about one half of the second volume, besides three chapters of the eleventh book contained in the fifth volume, and not translated by Bowditch. The difficulty of the various problems is reduced almost entirely to that of finding the sum of all the quotients obtained from dividing each particle of the spheroid

by its distance from a fixed point; and La Place has arranged this sum in a converging series of terms. The terms of this series are very remarkable in their properties and mutual relations, and, on account of their great utility in these and other inquiries, have been much noticed by geometers, and honored with the name of *La Place's Coëfficients*. Considerable objection has been made in regard to the generality of the results, which La Place obtained from them, because there are excepted cases to which they are not applicable. But Poisson has demonstrated, that they may be considered as embracing all cases which would practically arise; and the uninitiated may be satisfied, when informed, that the excepted cases differ but infinitely little from those which are strictly within the limits of the investigation, so that the error must be wholly insensible; and several formulas which are continually used in mathematics, without any question of their sufficient generality, are subject to precisely similar exceptions.

La Place's analysis, admirable as it is for its novelty and profoundness, is not altogether faultless. It has been exposed to repeated attacks, and contains two remarkable defects; one of which was not detected till thirty years, and the other not till nearly fifty years, after their first publication. The former of them was pointed out by Lagrange, in a theorem, which La Place had supposed to be applicable to every law, but which fails beyond a certain limit. And this limit is, by a singular coincidence, the very law of Nature; so that, if gravitation had decreased a little more rapidly, inversely, for instance, as the cube of the distance, the formula would have led to false conclusions. The second defect occurred in the fundamental formula of the whole calculation, and was much more recently discovered by Poisson, who observed, that La Place had run upon the very rock which he himself had surveyed, and committed the error against which he had expressly warned other mathematicians. But this error, vanishing in the course of further calculations, is, like the other, merely speculative; and it is probably on account of this very surprising fact, that they both remained so long unnoticed. For, with all its boasted accuracy and apparent independence of man's physical powers, it appears, from the frequent mistakes and disputes of its votaries, that geometry cannot avoid the defects to which the science of an imperfect

mind is ever liable ; and, requiring the promptings of observation to rouse and direct it to new efforts, it is slow to undertake the ungracious task of revising its labors, and detecting errors which have escaped the jealous eye of the practical astronomer.

Although the general form and attraction of the earth arise only from the gravitation of its particles, yet the composition of its different materials, and their character, as gases, fluids, or solids, depend upon the combination of an affinity or cohesion, which is insensible at perceptible distances, with the repulsive force of the caloric, with which each atom is surrounded. If the quantity of caloric is small, the repulsive force decreases more rapidly than the attraction, with an increase of distance ; and the molecules of a body, not being separated beyond the influence of their mutual attractions, are bound firmly together in the form of a solid ; and the only evidence of their want of contact is found in the compressibility or elasticity of the mass. By increasing the caloric, the particles are forced further apart, the body dilates, and, when they are so far removed as to cohere only by means of the attraction of each for the caloric of the other, they but slightly impede each other's motions, and La Place regards the body as reduced to a fluid state. But, as he observes, the surface must, in this case, be, at least for an imperceptible depth, much less dense than the interior of the body ; and be intermediate, in its character, between a liquid and a gas. The gaseous state is that, in which the quantity of caloric is so great as entirely to destroy the effect of the attractive forces ; and the fluid would soon be dissipated, if it were not restrained by the sides of the vessel which confines it. In this view of the subject, "all terrestrial phenomena depend," says La Place, "upon attractive and repulsive forces, which are only sensible at imperceptible distances, in the same way that celestial phenomena depend upon gravitation. I think, therefore, that the consideration of them should now become the great object of Mathematical Philosophy. It seems to me, that it would even be useful to introduce them into the demonstrations of mechanics, and banish the abstract ideas of lines without magnitude, and solids without porosity. I am convinced, by the trials which I have made, that, in thus conforming to Nature, these demonstrations might be pre-

sented with undiminished simplicity and increased clearness."

Within ten years of his death, analysis took the most gigantic strides, in the direction thus marked out by this great geometer; and Cauchy's wonderful success in developing the laws of molecular action has gained for him an immortality by the side of La Place himself. But the glory of first discovering this immense continent in science belongs to Newton, although Providence chose to check his daring course, and reserve some regions to be conquered by his successors; and the honor of erecting upon it the standard of Analysis must be given to La Place. He hence deduced the equations for the motions of a gas, and found them to differ essentially from the formulas before used, inasmuch as they contain the forces which arise from the heat, developed in different portions of the moving gas, and their increased density. These forces do not, however, sensibly affect the motions of masses of air, such as the oscillations produced by the attractions of the sun and moon; and their influence is only perceptible in vibrations like those of sound. In the Newtonian theory of sound, no regard was had to the effect of heat, in thus increasing the elasticity of air during its vibrations; and the velocity which was obtained did not agree with that derived from experiment. Newton, and most of his followers, attributed the difference to the presence of foreign particles floating in the air, while Lagrange supposed the observations to be inaccurate; but La Place was strongly convinced that these explanations were incorrect, and was not satisfied till he had discovered the true cause of the error. He applied his formulas to the motion of light, supposing it to consist in the undulations of the solar atmosphere, and obtained a velocity which was not one seven-hundredth part of the actual velocity of light. Hence he inferred, that, if the undulatory theory were correct, the ethereal fluid must be powerfully compressed; and, as no cause for such a compression is to be found in the celestial spaces, he would seem disposed to reject this theory. But Cauchy's conclusions are far more satisfactory. Discarding the necessity of any compressing force, he makes it evident, that this immense velocity is satisfactorily accounted for, by supposing the particles of ether to be millions and millions of times smaller and closer together than those of gas; so that, if density is

assumed to be proportionate to the number of particles in the unit of space, such an ether must be far more dense than the heaviest solid; but the exact reverse is the case, if the weight of the atoms are substituted for their number, in the definition of density. By combining an original and most powerful analysis with the happiest hypotheses, Cauchy has succeeded in establishing the wave theory of light upon a foundation not easy to be shaken. He has deduced from it the various laws of the propagation and polarization of light; and even the phenomenon of its dispersion, which had been regarded as an insurmountable objection to this theory, has not only yielded to his perseverance, but has become the strongest argument in its favor; since he has discovered the laws which regulate it, although they had been sought in vain by many great philosophers, and his results are most strikingly confirmed by their experiments. The theory of the emission of light has been regarded as consecrated by the favor of the founder of Optics; but this immortal genius was too often tormented by the galling attacks of his jealous rivals, to wander from the straight road of the Baconian philosophy, so far as to adopt such an unnecessary hypothesis. As an instance of his care, even in the use of words, he would not introduce the term "attraction" into his "Principia," without a caution to the reader, "Caveat lector, ne per hujus modi voces cogitet me speciem vel modum actionis, causam aut rationem physicam alicubi definire, vel centris (quæ sunt puncta mathematica) vires vere et physice tribuere." He observed facts and generalized them. Not suffering his fancy to direct his reason, his deductions are almost as indisputable as his observations, and his theories must endure as long as the sciences to which he gave birth.

Another case of corpuscular action, which was investigated by La Place, and is "remarkable from the variety and the singularity of the phenomena depending upon it," is, that of Capillary Attraction. Though Newton's wonderful instinct had led him to divine the laws which regulate some of the capillary phenomena, Clairaut was the only geometer, till the time of La Place, who had reduced them to a rigorous calculation. His great error, however, and one which he would have discovered, if he had extended his analysis to other cases, consisted in supposing the molecular forces to be sensible at perceptible distances. By avoiding this error, La

Place succeeded in explaining all the phenomena with the most rigid accuracy; and the most peculiar and striking point of his theory is, perhaps, that he makes it chiefly depend upon a circumstance which had been altogether disregarded, and shows, that the concavity or convexity of the surface of a fluid, contained within capillary spaces, instead of being a mere secondary effect of the corpuscular action, is the principal cause of the rise of the fluid. This view of the subject is, however, a purely mathematical one; and a juster physical statement of it, corresponding, in our opinion, almost exactly to the ideas of Newton, and introduced into La Place's second Supplement, is, that the particles at the surface of the fluid in contact with the capillary walls, being raised above their level by the attraction of the walls, exert their attraction again upon those which are near them; and this action continues until the weight of the elevated fluid balances the attracting force, which tends to raise it higher; but, as the uppermost link of a suspended chain must bear the whole weight of the chain, so the weight of the raised fluid must be proportionate to the elevating power of the attractive force exerted by the walls upon the adjacent particles, that is, to the horizontal circuit of the walls. Vague reasoning must not, however, usurp the place of accurate calculation; the difficulty of solving the capillary problems may be better understood by observing, that they depend upon the very principles involved in determining the figure of the earth; and we may remark, that we think Ivory would find no little difficulty in applying to this case his new principle.

"In order," says Mr. Pickering, in his noble Eulogy before the American Academy, "that the importance of this subject may be understood, and that a just view may be taken of the extent of it in its various relations, we must reflect for a moment upon some of the numerous modes, in which this species of attraction exhibits itself.

"The most usual form in which it has been the subject of observation and experiment, is, in the ascent of water, or any other fluid, in *capillary* tubes, or between two plates of glass placed near each other in a vessel containing the fluid. The same principle, however, governs the movements of fluids in numberless other cases; some of which are so familiar to us, that they cease to attract our notice. For example; when we fill a glass or other vessel with water, if the vessel is already wet, the water will be drawn upwards round the sides of the vessel,

and present a *concave* surface ; but if, on the contrary, the vessel is entirely dry, the water will rise in it with a *convex* surface, and may, in popular language, be heaped up even above the brim of the vessel. From the same cause, a light body floating on the water near the side of the vessel, will suddenly be drawn into contact with it ; and two bodies lying on the surface, upon being brought towards each other, will suddenly rush together. In the same way, too, we see the rain forming itself into pellucid drops, and hanging from the under surfaces of bodies, or standing in imperfect globules on their upper surfaces ; and the same principle manifests itself in the form of

> 'the dew-drops, which the sun
> Impearls on every leaf and every flower ;'

and in

> 'the gentle tear let fall
> From crystal sluce.' *

"In short, the phenomena of capillary attraction are so constantly manifesting themselves, and under such various circumstances, that they present to the philosophical observer questions of singular interest and extraordinary difficulty. These questions are most elaborately and profoundly investigated by La Place and his commentator.

"Among other investigations of Dr. Bowditch in relation to this subject, I ought not to omit the fact, that he has most thoroughly examined and analyzed the very celebrated work of the present day called the *New Theory of Capillary Attraction*, by the eminent French mathematician, M. Poisson ; and has shown, by numerous examples from M. Poisson's work, that, profound and acute as that author is, he has, under a different form of notation and with vast labor, only arrived at results which are either identical with those before obtained by La Place under his own form of notation, or which may be easily obtained from them ; and that the supposed discoveries announced in the *New Theory* have not in reality advanced this branch of science. This portion of Dr. Bowditch's work, when published, will, in the opinion of our mathematicians, attract the notice of men of science in Europe as strongly, perhaps, as any part of his labors." — pp. 72 – 75.

Almost the only essential difference between the theories of La Place and Poisson arises from the consideration of the diminution of density near the surface of a fluid, which is neglected by La Place, as of no importance, and expressly introduced into the calculations of Poisson.

" * Milton's 'Paradise Lost.' "

"This change of density produces a corresponding change in the value of the capillary intensity; but as this quantity can be found only from actual experiments, and not from the analytical expression, it leaves the results of La Place's theory unimpaired in all the formulas depending on this quantity."—Bowditch's *Translation, &c.*, Vol. IV. p. 687.

Poisson attempted to render his researches much more general than those of La Place; he was, however, compelled to omit term after term, until the limits of "the new theory" were narrowed down to those adopted in the "*Mécanique Céleste.*"

Returning, then, to the earth's figure, we shall pass over the small inequalities of its surface to the still smaller irregularities of figure arising from the undulations of the ocean. How minute these are, compared with the magnitude of our globe, we can conjecture from the fact, that the ocean itself is but as the moisture of the shower upon the surface of the apple. The idea of attributing the ebb and flow of the sea to the influence of the sun and moon originated with Kepler, but was first subjected to calculation by Newton, who supposed the sea to assume, at each instant, the form of perfect equilibrium. This feature of his theory was adopted as the basis of all calculations upon the subject, till La Place considered the problem in its true light, and applied to it the general principles for determining the motions of fluids. But the impossibility of completing his analysis, by the aid of the limited observations which he obtained, compelled La Place to leave it in an imperfect state; far superior, however, to the contempt with which an English philosopher has thought proper to visit it.

"La Place's solution," says Whewell, "besides being obtained by means of a precarious assumption, rests upon several arbitrary hypotheses, fatal to it, even as a first approximation; and, I believe it will be found, leaves out of consideration an essential portion of the forces. To obtain any useful result, the question must be taken up afresh and treated in another manner. I hope some mathematician will be found able and willing to execute this task. But, in the mean time, I may be permitted to observe, that what has been already done in the discussion of tide observations, and in bringing to light the empirical laws of the phenomena, has entirely altered the position of this branch of science with respect to the mathematical theory. A little while ago the theory was in advance of ob-

servation; at present observation is in advance of theory. A very few years since, the equilibrium theory and the La Placian theory were in a condition to assign laws regulating the changes of the times and heights under given astronomical circumstances, and it had not been shown from observation whether these laws were obeyed. We can now state what the agreement and disagreement are between such theoretical laws and the facts; and we call upon the mathematician to substitute for these two theories, both confessedly false, some other, which shall come nearer to the true state of the case, and, by that means, nearer to the laws of the phenomena. The performance of this task is requisite for the completion of the Newtonian theory of the universe."

Notwithstanding this sweeping condemnation, we cannot cease to look upon La Place's theory as one of the finest monuments of his genius, and as being fundamentally correct. Judged by the narrow standard of its discrepancy with observations, it undoubtedly is deficient; and, by the same rule, Newton's lunar theory would have merited the reproaches of Flamsteed, instead of the generous admiration with which it was regarded by La Place. The subject of the tides does, indeed, require a fresh examination, in order to satisfy the recent observations due to the efforts of Whewell, for which, and for his successful exertions in reducing them to empirical laws, he richly deserved the medal of the Royal Society. The equilibrium theory, as developed by Bernouilli, may, likewise, be better adapted, from its simplicity and neatness, to the mind which is seeking for these laws, and has not the comprehensive grasp, or the mathematical power, to enter upon a profound analysis of one of the most complicated questions of astronomy. The additions which it is continually receiving, must, however, soon render it too cumbrous a machine even for this purpose; and geometers will be compelled to return to the direct course of investigation, as first explored by La Place.

Were the earth a perfect sphere, its equator would always cut the heavens in the same plane, and its axis be directed to the same polar star; and if the polar axis were to be assumed as the diameter of this sphere, the excess of the terrestrial spheroid might be detached from it, and collected in a ring at the equator. The attraction of the sun would draw the plane of the ring from that of the equator, and the points of its intersection with the ecliptic would retrograde contin-

ually, according to a law easily determined from the similar motions of the nodes of the lunar orbit. Such a backward motion, though much retarded by the mass of the earth, which the ring must carry with it, is still sensible, and constitutes the great phenomenon of the precession of the equinoxes. The attraction of the moon produces a similar effect; and there are also some inequalities in the phenomena, which have been thoroughly investigated by D'Alembert, Euler, and La Place.

If we now proceed from our own globe to the other bodies of the solar system, we find them all, so far as we can observe them, possessed of oblate figures, corresponding to their respective velocities of rotation, and moving in circular orbits about the central source of light and heat. Upon a careful examination of their motions, it is found that they are not uniform, nor exactly circular; and we admire the genius which detected, amidst their great inequalities, an average and permanent regularity. From Kepler's celebrated laws, to derive the law of gravitation, would appear a most simple application of the Fluxions of Newton; so simple, indeed, that the mathematician of the present day can only estimate the magnitude of the stride, from the number of gigantic minds which could not accomplish it. But the law of gravitation, once established, was soon found to contain within itself a perfect and entire system, altogether above the approximations of Kepler, and embracing all the complicated phenomena of the heavens, while the latter would only be strictly true if the planets exerted no influence upon each other. Drawn by their mutual attractions, however, out of their elliptical orbits, the planets so disturb each others' motions, that the exact determination of their motions is no longer possible. Theory is forced to lower its aim, and endeavour to arrive at approximations, whose inaccuracies may be so minute, that the nicest observer cannot detect them. To arrive at this point, which has not by any means been universally attained, it has been found sufficient, in most cases, to calculate, separately, the effect of each small disturbing force; and thus, the perturbations arising from each planet cannot be confounded with the others. By this means the problem is evidently simplified, but still not enough so to admit of a complete solution; and the most delicate tact is required, to choose the most rapid system of approximation, and to

select, among an infinite mass of terms, those which are of sensible magnitude.

The difficulty of this selection is much increased by the facts, that terms originally diminutive, will sometimes in the course of the calculations, acquire equally minute divisors, so that the quotients may be quite too large to be rejected. One of the most striking instances of such an effect is observed in the reciprocal perturbations of Saturn and Jupiter. The mean motions of these two planets are almost commensurable, five times the mean motion of Saturn being nearly equal to twice that of Jupiter; so that the difference between these two products, being very small, augments considerably the terms of which it is a divisor, and the corresponding inequalities, instead of being almost insensible, are the most important of all those observed in the motions of these planets. A tolerably correct physical explanation of this phenomenon is derived from the consideration of the great effect produced by a comparatively small force acting constantly, at the most favorable intervals; a familiar, though homely example of which, may be found in the often ludicrous difference between the skilful and unskilful attempt to turn up the heavy church bell to its unstable position of equilibrium. By discovering the simple ratio between the mean motions of Jupiter and Saturn, La Place was enabled to account for those irregularities in their motions, which had before seemed quite inconsistent with the law of universal gravitation, but are now regarded as one of its most striking proofs. "Such," says La Place, "has been the lot of Newton's brilliant discovery, converting every obstacle into a new source of triumph; the surest characteristic of the true system of Nature." If the mean motions of these planets had been exactly commensurable, their perturbations would have been greatly increased, and they would have presented a phenomenon of the highest analytical interest, and of which the solar system contains but a single instance. This might easily have been the case; for, if the small variation from perfect commensurability had been a little less than one half its present value, the mutual attractions of the planets would soon have caused it to vanish entirely.

Jupiter forms, with his satellites, a solar system in miniature; and the frequent eclipses of the satellites enable the observer to determine their orbits with considerable accuracy,

and to watch their progress through all those changes, which the planets take ages to accomplish. A thorough analysis of their motions, which was first undertaken by Lagrange, was remarkable, as not being limited to the separate consideration of each disturbing influence, but as combining them all together; "a point," says Bowditch, "of the utmost importance, since upon it depends the law regulating the motions of the three inner satellites." These laws, called La Place's, because he first detected them, are of the class of commensurabilities between the mean motions, which are so important in the theory of Jupiter and Saturn; and they contain the single instance, but just alluded to, of a perfectly exact ratio.

Among all the perturbations of the celestial motions, none, however, are more interesting to the terrestrial observer, than those of the earth's satellite; and they are much augmented by the powerful disturbing influence of the sun. The few of them which Newton had succeeded in calculating, were enough to confirm, in a most important manner, the law of gravitation, though the great bulk were left unexplained; and even now there are several points in the lunar theory, about which the minds of geometers are not at rest. The difference between the values of the same terms, as given by each of the great astronomers, La Place, Plana, Lubbock, and Poisson, is sometimes most remarkable, and seemingly inconsistent with the admired accuracy of mathematical calculations. Thus one equation, determined from observations by Damoiseau to be greater than six hundred seconds, was at first reduced by the computations of Plana, to less than five seconds, though he afterwards raised it to a value which somewhat exceeded that found by Damoiseau. But even such a failure of theory must not be regarded with uncharitable surprise and severity; "for," as Lubbock remarks, "when we consider the enormous number of terms which are necessary to be taken into account, and how difficult it is altogether to avoid error in numerical calculations, it is hardly to be expected, that the result can be entirely free from error."

Most of the numerous inequalities of the lunar motions have been deduced from observations before they were recognised by theory, and analysis has often been sadly perplexed to explain them. One of the most singular instances, is that of an equation inserted by Mason, in his Lunar Tables, which seemed so entirely inconsistent with the laws of gravitation, that it

was originally rejected by astronomers, although they were ready to admit the superior accuracy of his tables in other respects. From a happy application of his favorite theory of probabilities, La Place decided, that the argument in favor of the existence of such an equation was so strong, that he resolved to seek its origin. His search was successful. He found it to arise from the oblateness of the earth; and his investigations led him to the discovery of another similar inequality, which had escaped the notice of observers, though indisputably confirmed by later observations. He also reversed the problem, and made use of the equation as the most accurate means of determining the oblateness. But analysis is thought not always to have been so successful, and a single equation is still adopted by some observers, which remains wholly unexplained. Its period is a very long one, and is not exactly determined. Poisson has demonstrated, that it cannot arise from the direct action of the sun or of the planets upon the moon; nor from the action of Venus upon the earth, which produces, as Airy has shown, a sensible equation in the earth's motions; nor from the terrestrial oblateness or irregularity of figure. He, therefore, doubts its existence, and regards the observations from which it is deduced, as either too recent, or too inaccurate, to be depended upon for determining so long an inequality.

No more important question has ever agitated the minds of astronomers, than that regarding the perfect accuracy of the law of gravitation. Bessel seems to consider it as a most doubtful point; for he found observation not unfrequently to differ from calculation, by five seconds or more; quite too much, as he says, for an error of observation; full as much, indeed, in the present state of science, as were the eight minutes, at the time of Tycho Brahe, which gave Kepler the means of reforming the whole of astronomy. But the history of astronomy is filled with such doubts, which have all vanished before the progressive advances of analysis; and so intimate is the connexion between theory and observation, that they must for ever be, alternately, taking the lead in the march of science. The earlier theories of the moon were defaced by errors of not less than seven or eight minutes; and, though they are now reduced to as many seconds, it was actually proposed by Clairaut to introduce, into the law of gravitation, a term, which should vary with the inverse cube of

the distance ; but he himself soon proved, that this term was not required. Again, it was found, that the mass of Jupiter, as deduced from his attractions upon Pallas, and upon Encke's comet, was quite different from that derived from the distance of his satellites, or from La Place's theory of the perturbations of Saturn. Such a difference could only be accounted for by supposing the attractive power of Jupiter to be less for his satellites and for Saturn, than for the comets and the telescopic planets ; and the result of this hypothesis would be, that the power of gravitation varied, like the chemical affinities, with the nature of the different attracting substances. But, from a new and more correct measurement of the distances of the satellites, a result has been obtained, which agrees almost exactly with that derived from Pallas and the comet ; and, upon a review of the theory of Jupiter and Saturn, Pontecoulant has most unexpectedly found, that the chief term of the great inequality was taken by La Place with the wrong sign, and the correcting of this error has given the same value of Jupiter's mass as that otherwise obtained, and removed all the apparent anomaly. The arguments, indeed, in favor of the perfect uniformity of the gravitating influence are founded upon so many nice experiments and accurate calculations, as to be almost irresistible. Not the slightest difference can be detected in the attractive power of the earth for the moon and for any of the substances found upon its own surface, by means of the most delicate experiments made with pendulums, composed of every variety of material. The distance of the sun can be computed with so great accuracy from the motions of the moon, that the difference between the attractive powers of the sun for the earth and its satellite, cannot be the one-millionth part of the whole force ; and a similar result with regard to all the substances of the solar system may be deduced from the universal agreement of observations with the theory based upon the law of gravitation.

Another interesting inquiry concerns the stability of the solar system. The planets move, at present, in orbits nearly circular, and almost in the same plane ; and La Place professed to have demonstrated, that this would always be the case. But Lagrange and Bowditch* objected to his demonstration, considering it to be "satisfactory for the three great

* See, on this subject, an article from the pen of Dr. Bowditch, in the *North American Review*, Vol. XX. pp. 309 *et seq.*

planets Jupiter, Saturn, and Uranus, but not for the other planets, whose orbits, for aught that is proved to the contrary, may be very eccentrical," and depart very much from the same plane. But these eminent philosophers have, in our opinion, taken a partial view of La Place's argument, and have not done it full justice. They have, inadvertently, passed over his first step, which will be found, upon a close examination, to be of essential importance ; and in which he proves, that, if there be any limit whatever to the increase of the eccentricity of either of the planets, it must be a very small limit ; so that the only object of the remainder of the demonstration, is, to prove that there is a limit, without any regard to its magnitude, as the objections appear to imply. Viewed in this light, his reasoning seems hardly to admit of dispute. But we think that, in the arguments upon both sides, the physical restrictions of the question have been too much overlooked. Thus it is said, that "the orbits may become very eccentrical, or even parabolic." But how is it possible, by any increase of eccentricity, for an ellipse, whose greater axis has been long before proved to be invariable, to degenerate into a parabola, which has an infinite axis ? On the contrary, when the ellipticity augments so as to become unity, the orbit, instead of changing into a parabola, would be reduced to a straight line, terminating in the sun ; and, by a further increase of ellipticity, the orbit would become imaginary, unless the planet happened to be at one of the extremities of its orbit, at the precise instant of this increase, against which the probability is infinitely great. It is, therefore, almost physically impossible, for the eccentricity to become greater than unity, and the second part of La Place's demonstration seems to be nearly superfluous.

But there is a force in constant operation, which is directly opposed to the eternal duration of the solar system. The constant resistance of the ether or the light which pervades all space, small though it be, must at length destroy the motions of the planets, and cause them to fall into the sun. Ages, whose vast extent and number the mind of man cannot comprehend, must indeed roll by, before such an effect can result from so slight a cause ; and yet it is as inevitable as the wearing away of the stone under the incessant dropping of the water. And that there is such a sure principle of destruction in our world, affords an irresistible proof, that the

system cannot have existed in its present form beyond a certain time; but it must have had a beginning, a creation, and a Creator.

It might, however, be doubted, whether the ether were a resisting medium, if it had not been demonstrated by the retardation of Encke's comet. The comets, which fly in every direction around the sun, are so very light, that they have not, in any appreciable degree, disturbed the motions of the planets, though the fact that they are not destitute of weight, and are subject to Newton's great law, is obvious from the form of their orbits, and the great perturbations which they have suffered, from the planetary attractions. Thus the time of revolution of the comet of 1770 was diminished more than two days by the influence of the earth, and its orbit was so entirely changed during its near approaches to Jupiter, in 1767, and 1779, that it probably never was visible from the earth before that time, and never will be again; and yet its reciprocal action upon these planets was wholly insensible, and it passed directly amongst Jupiter and its satellites, without having produced the slightest alteration in their motions. The computation of the orbits of the comets, originating with Newton, has been most successfully developed by the labors of Lagrange, Legendre, La Place, Olbers, Gauss, Ivory, and Lubbock; and their various methods have been collected by Bowditch into the splendid Appendix to the third volume of his translation. He has there given most of the great improvements made by these geometers, "in the calculation of the orbit of a planet or comet, moving in an ellipsis, parabola, or hyperbola," and he has added more than one hundred pages of beautifully printed auxiliary Tables, to lighten the task of computation; "some of which are new, the others have been varied in their forms, to render them more simple in their uses and applications, and none of them have heretofore been published in this country." In the commentary upon the tenth book, contained in the fourth volume, Bowditch gives the calculations, which have been made regarding the retardation experienced from the resistance of the ether by Encke's comet, the result of which is, that "the time of its periodical revolution, which is a little over twelve hundred days, will be decreased about one-tenth of a day in each revolution; and the mean distance will be decreased about one sixteen-thousandth part."

Leaving, now, the solar system, of which we are but insignificant particles, we will travel to the glorious suns, which are diffused throughout all space. Passing over their differences of color and splendor, with their remarkable periodical changes, and that sudden appearance and disappearance of others, which indicates the existence of opaque celestial bodies, perhaps not smaller or less numerous than the visible stars themselves, we proceed to a phenomenon, at first sight of much less apparent interest, the grouping of the stars together in immense clusters, which are separated from each other at distances prodigiously greater than the dimensions of the cluster. The group to which we belong, consists of the Milky Way, and the stars immediately around us. Among the other clusters, some are so remote, that the most powerful telescope does not enable the observer to distinguish the different stars, while the cloudy appearance of others indicates an imperfect state of formation; and the elder Herschel has traced their progress from the state of mere nebulæ; not that he has witnessed a change in any single case, but "just as in a forest the growth of the trees may be traced by means of the individuals of different ages." He observed, in one part of the heavens, a mass of mere nebulous vapor, of uniform density; in another, this vapor was feebly condensed about a slightly brilliant point, or about a multitude of such points; then, again, the points increased in brilliancy until the condensation was at last carried so far, that the atmosphere of each nucleus separated from the rest; and multiple nebulæ were formed, consisting of brilliant nucleuses, surrounded each by an atmosphere. The condensation still went on, until at last a sun was formed, surrounded by an immense atmosphere, or a set of two or more suns revolving about each other like the double, triple, &c. stars, or a group of suns, such as are the Pleiades. This was the state of our system, ages and ages before the commencement of the Mosaic creation, and its probable progress during this immense interval, first developed by La Place in the "*Systême du Monde*," is the most magnificent conception that was ever formed in the mind of a philosopher. He chose, however, to diffuse around it the foul vapors of atheism, and it is now dreaded by the Christian sage, like the proud city amid the miasmata of death.

The substance of the sun could hardly have separated from those of the neighbouring suns, without acquiring some small

rotary motion, and, as its dimensions lessened with its increasing density, the rotary velocity must also have increased, by the known laws of dynamics. It is thus, that the angular velocity of a sling is rapidly augmented by allowing it to wind itself about the finger, or that the rotary motion given to the water is greatly accelerated as it flows through the small hole in the bottom of a cistern. The sun must, at last, turn so swiftly, that the centrifugal force of the outer zone, overcoming the gravitation, must cause it to separate from the inner mass, and form a ring about the sun, like that of Saturn. By further condensation, new rings are successively thrown off, each of which gradually draws together its particles into a nucleus and forms a planet; or, if there are several nucleuses, there must be as many smaller planets, which, like Ceres, Pallas, Juno, and Vesta, will be at nearly the same distance from the centre of motion. Since the particles of the ring's inner surface are nearer the sun, their velocity must be less than those of the outer surface, so that when the planet is formed, it must have a rotation, which, increasing as the planet condenses, may cause it to part with its own rings, and form its own satellites.

This grand but simple view of the process, which the Deity has adopted in framing the universe, bears upon its front the stamp of truth and of divinity; it necessarily leads to a system, which the law of gravitation renders permanent; and it accounts for all the leading phenomena of astronomy. Hence it is, that the planets and satellites revolve and rotate in the same direction, and almost in the same plane; that their orbits are nearly circular; and that their densities are less the further they are from the sun. Hence, too, it is, that the satellites always turn the same face to their primaries; for if, after the formation of a satellite, it were not to condense, its time of rotation would be exactly the same with that of its revolution; and the extremely gradual condensation to which it is liable, cannot prevent its assuming such a form under the attraction of the primary, that this attraction must in future restrain the rotary motion, and, compelling it to turn its elongated face to the primary, will only allow it to be affected by the phenomenon of libration. The disturbing influence of the unequal density of different parts of the nebulæ must affect the whole course of these phenomena, particularly in the earlier stages of the formation, and must cause such irregularities as these; that the

density of Uranus is greater than that of the two next inferior planets ; and the planes of its equator, and of the orbits of its satellites, are nearly perpendicular to that of its own orbit. If, when a zone separated, indeed, it were more dense on one side of the sun's equator than on the other, the planet's rotation and revolution would be performed in different planes ; and a little reflection will show, that the inclinations of the orbits would be the most affected by such a cause in the case of the inferior planets, and those of the equators in the superior planets.

The comets, in this system, are themselves smaller nebulæ, with nucleuses of their own, which have come near enough to the sun, to be subject to the laws of its attraction, and revolve in very eccentrical orbits ; and, as there is no general reason to influence the primitive directions of their motions, we find their orbits inclined at every variety of angle to the sun's equator. "If, finally, in the zones successively abandoned by the solar atmosphere, there are atoms too volatile to unite with each other, or with the heavenly bodies, they must continue to circulate about the sun, and offer all the appearances of the zodiacal light, without opposing any sensible resistance to the planetary motions."

But inquiries regarding a state of the world, so far removed from its present one, are of no practical value, and must rank with those questions of curiosity, upon which much labor has been expended by mathematicians. And a similar remark may be extended, with some qualification, to almost every part of the "*Mécanique Céleste*," or of the "Principia." For, humiliating as is the confession, it must be admitted, that practical astronomy is not dependent upon theory, and that the observer, with only a small degree of elementary mathematics, can deduce from his observations empirical laws sufficient for all his purposes. Judged, then, by the narrow standard of utility, which is generally adopted, and has been pronounced to be at the very foundation of the Baconian philosophy, transcendental mathematics can offer but little to redeem it from the reproach of being a misletoe science, uselessly absorbing the highest energies of the most powerful minds. Even its noblest fruit seems to be blasted ; its Newton was sensitive, fretful, and almost mean ; and its La Place was an atheist.

But the character of the votary must not condemn the sci-

ence ; and the character of the science must be referred to its true standard, that of knowledge. If we would, then, search deeply into the laws of nature, we find, that the inner doors of physical philosophy must be opened with the keys of geometry ; and that even the trials of skill, and the mere questions of curiosity, in which mathematicians have delighted, are the best possible preparation for the more serious contests of science, and have again and again led the way to the most brilliant investigations. It was thus, that Lagrange reduced the whole of Mechanics to a problem of Isoperimetry, and made its solution depend upon an analysis, which had before been admired and prosecuted only as an exercise of geometrical gymnastics, on account of its singular beauty, and the remarkable simplicity of its results. As far, indeed, as the great object of life and of knowledge, the improvement of the mind, is concerned, the humblest subjects may furnish inquiries no less useful than the most sublime. The path of the arrow is as difficult of calculation as that of the comet, and the celebrated problem of the precession of the equinoxes is of the same class with that of the spinning of the top.

What difference is there, then, between the intellectual importance of the investigations, except that the planet and the comet are observed with care, while the top and the arrow are the playthings of a thoughtless child ? And what difference does it make to the mathematician, whether the data of his problem are taken from the external world, or from the creative power of his own mind ? Suppose that the telescope had never been invented, and that there had never been any means for making accurate observations of the heavenly bodies ; and suppose that, as a subject of intellectual exercise and curiosity, the mind of Newton, or of La Place, had devoted itself to the calculation of what would have been the motions of a system of bodies united by the law of gravitation ; would the labor have been less worthy of his genius, and less fitted to develope his mental powers ? And suppose that he had prosecuted his investigations still further, and, applying his analysis to other laws of attraction than that of nature, had attempted to discover what law and what arrangement would have led to a permanent system, free from all great irregularities ; would not this mere question of curiosity have deserved the attention even of such a mind ? Could any intellectual exercise be thought of, more nearly allied to that

required at the creation? Is it not the very labor which would absorb the powers of the archangel, to whom should be intrusted the sublime task of building a world? Such an inquiry has actually been instituted by geometers, and they have almost established the proposition, that, with none but the present law of gravitation, and with no other arrangement, would the solar system have been permanent. We regard this grand result as alone worthy of all the time, and all the genius, which have been devoted to mathematics.

Dr. Bowditch was able to superintend the passage of his great work through the press, as far as to the one thousandth page of the fourth volume, his disease (a *schirrus* of the stomach), though at times extremely painful, not having compelled him wholly to intermit his studious labors till within a few days of his death. That event took place on the 16th day of March, 1838, when he had nearly completed his sixty-fifth year. Tributes of respect for his services and character were offered from numerous quarters, on the part of corporations and societies of which he had been a member, and of various learned bodies. The American Academy, the President and Fellows of Harvard University, the Faculty of Yale College, the Trustees of the Boston Athenæum, the Massachusetts Life Insurance Company (of which, from its foundation, he had been the executive officer), the East India Marine Society, and other associations, held meetings, and passed resolutions expressive of their sympathy with the bereaved family, and of their sense of the public loss. Measures were taken for erecting a monument to his memory; and eulogies were pronounced by the Reverend Mr. Young, in the church where he habitually worshipped; by Judge White, at the request of the Corporation of Salem, of which city he was a native, and during most of his life an inhabitant; and by Mr. Pickering, before the American Academy, of which, for several years and down to the time of his death, he was the presiding officer.

These Eulogies treat at large the life and character of their subject, and preserve, what posterity will gratefully receive, a trustworthy and full description, both of the philosopher and of the man. The learned oration of Mr. Pickering embraces a biographical outline, but is chiefly remarkable for its thorough and luminous analysis of Dr. Bowditch's scientific

publications, exhibiting, in a comprehensive and discriminating sketch, the nature and value of his labors, as well as the actual condition of that sublime department of study, on which they were employed. The discourse of Judge White, — a composition of rare excellence in its kind, — traces, step by step, the progress of this great mind, satisfactorily indicating the stern but effective discipline under which it was trained, the traits which characterized its eminence, the principles which controlled and gave success to its action, and the variety of beneficent influences exerted by it upon society; — subjects which, in the eulogy of Mr. Young (the earliest and most copious of the three), are also felicitously treated, and illustrated by a large collection of interesting facts and anecdotes.

We have not proposed to ourselves to offer so much as a brief sketch of the life and character of this distinguished individual, thinking it to be rather the office of a work like ours, to dwell upon his services to science. But, while there are few who will read his writings, there are many who may profit by the record of his virtues, and of the principles and habits which contributed to his greatness; and, little inclined as we are to the formal inculcation of a moral, we yet find ourselves unwilling to dismiss the subject, without giving a few words to some of those facts and traits, which, so judiciously exhibited in the biographical notices we have specified, make them fruitful of the best instruction. It was from an humble condition in early life, that (in part, no doubt, by force of extraordinary natural endowments, but also by force of a principled energy, alert to take advantage of every opportunity of improvement, and refusing to be depressed by any discouraging circumstances,) Dr. Bowditch rose to be one of the most eminent persons of his country, and of the time. The son of a working cooper, enjoying no advantages of instruction in early childhood beyond those of attendance on a public school, and those only till he was ten years of age, he was, two or three years after, apprenticed to a ship-chandler, and continued in this service through his minority; at the end of which time he went to sea, as an inferior officer in a merchant vessel. Meanwhile, by the diligent use of such fragments of time as he was able to redeem for study, from regular daily employment of so different a kind, he had, (besides laying up stores in general literature, which would have

done no discredit to a youth devoted to that pursuit) made such proficiency in his favorite science, as enabled him, three years after, to publish a work, the "Practical Navigator," scarcely surpassed in usefulness by any of the time, and immediately driving all others of the same class out of circulation. Being unable to purchase books, he borrowed and copied such as he most needed, possessing himself thus, before he was fourteen years old, of a long treatise on Algebra, another on Geometry, and a third on Conic Sections. At fifteen, making all the necessary calculations, he had arranged an Almanac, complete in all its parts. Obtaining, by a fortunate accident, a copy of Newton's "Principia," he learned Latin by himself, that he might read the work, and made a translation of the whole of it.

Entering upon an active life of business, Dr. Bowditch made four voyages to the East Indies, and one to Europe, diligently devoting his leisure at sea to his favorite inquiries, which, however, with a liberal sense of the value of other knowledge, he diversified by studies of a more generally attractive kind. Retiring from a seafaring life, at the age of thirty, he assumed an office, that of President of an Insurance Company in his native town, which, to most men, would have seemed to afford sufficient employment for their time; and from this, at the end of twenty years, he was transferred to the place of Actuary of the Massachusetts Life Insurance Company, which he held till the time of his death. It was by an economy of the leisure hours of a life thus engaged, that Dr. Bowditch won for himself one of the highest names in science, which the nineteenth century boasts.

Nor was it by any jealous and churlish economy of those hours. No man acknowledged more readily the claims of friendly intercourse; no man welcomed more cordially the interruptions which they bring. His study was his parlour, where no posture of a hard, unfinished problem ever caused the unexpected guest to feel, that his visit was untimely. No abstraction ever revealed the toiling or wearied mind. A gay buoyancy of spirits, and a prompt interest in whatever subject was presented, showed, whenever you found the man, that you found him before his work, and at his ease. Early hours, an utter abstinence from mere waste of time, and temperate habits which preserved the mind in perpetual vigor, permitted a life crowded with labor and its fruits to be, in

an equal degree, tranquil, free from care, and accessible to incidental engagements.

Along with great heartiness, Dr. Bowditch had its usual attendant, a warm impetuosity of character ; and, though no "rude and boisterous captain of the sea," there may have been occasions when a happier combination would have been produced, had the same measure of the *fortiter in re*, been blended with more of the *suaviter in modo*. But his high and rigid integrity was beyond question. His punctilious justice in the conduct of complicated affairs was a model for imitation. If he had prejudices, he had candor to welcome and weigh the evidence which would dispel them ; and anger he carried "as the flint bears fire" ; the spark was quick, but it was momentary.

Acquiring what in a frugal community may deserve to be called wealth, he had the high wisdom to know its worth ; that is, to know its uses. He cared for it as making him independent, and enabling him to be useful. In his life, as well as at his death, he gave freely from it to worthy objects of benevolence, public and private ; and he expended a large portion of it, without any hope of remuneration, on the publication of his great work ; declining, from a nice sense of honor, the urgent proposals of a learned society (the American Academy), and of private friends, that he would permit it to be issued at their charge. Of his time, his counsels, and his influence, he was as liberal, for good objects, as of his money.

Proof against less mischievous delusions, the madness of the "undevout astronomer" had no place in his clear and sober mind. The Christian faith, the support of his principles through a long, active life, was a sufficient source of consolation to him during the well-understood approach of death. Of cant and pretension, no man ever had less. But he had as little respect for the affectation which suppresses and disguises cherished sentiments, as for that which obtrudes and parades them. He thought it due to the truths which sustained him, to allow it to be known, that it was on them that he leaned ; and the chamber of his decline was a scene of the sublimest instruction for whoever would know, with what serene, magnanimous satisfaction, the spirit, which has well done the first part of its work, may pass on to its higher destinies.